MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2022/2023

DMS5131 – MATHEMATICAL & STATISTICAL TECHNIQUES

(For DIT students only)

03 JULY 2023
02.30 pm – 04.30 pm
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of **2** pages with **3** questions. Key formulae are given in the Appendix.
2. Answer **ALL** questions.
3. Write your answers in the answer booklet provided.
4. All necessary working steps must be shown.

Question 1

a) Given that $\lim_{x\to 1} p(x) = 2y, \lim_{x\to 1} q(x) = -8y, \lim_{x\to 1} r(x) = y + 4$. Find the value of y if

$$\lim_{x\to 1} \frac{p(x) - 2q(x)}{r(x) + 8} = -2.$$ (2 marks)

b) Evaluate $\lim_{x\to -\infty} \frac{3x^2 + 2x^3 - 4x^4}{x^3 - 4x^6}$. (1.5 marks)

c) Differentiate $f(w) = \frac{7}{\sqrt{w}} + \frac{\sec w}{5} - \frac{5e^{4w}}{4}$ and express your answer in **positive exponent.** (2 marks)

d) Differentiate the function below by using **implicit differentiation**. Express your answer in terms of $\frac{dy}{dx}$. (4.5 marks)

$$e^{y}\cos x + 2y^2 = x^4 - \cot y$$

[TOTAL 10 MARKS]

Question 2

a) Evaluate $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(2y^3 + \cos y\right) dy$. [Note: **leave your answer in terms of π**]

(2.5 marks)

b) Solve $\int_0^1 \frac{2x}{\sqrt{x^2 + 8}} dx$ by using the **Substitution Rule** and express your answer in **4 decimal places.** (4.5 marks)

c) Integrate the indefinite integral $\int \left(\frac{3}{2}x - 7\right)\cos x \, dx$ by using the **Integration by Parts.** (3 marks)

[TOTAL 10 MARKS]

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Question 3

a) Given the first order differential equation $\frac{dy}{dx} = ye^x + 2y$,

i) Solve by using the **separable method**. (2 marks)

ii) From your answer in part a(i), determine the solution of the initial value problem if $y(0) = 1$. (1.5 marks)

b) Given the differential equation $x^2 \frac{dy}{dx} + xy = x\sin x + 3x^3$,

i) Find $p(x)$ and $q(x)$. (1.5 marks)

ii) Solve for y by using the **method of integrating factors**. (3 marks)

c) Find the **particular solution** of homogeneous equation $y'' - y' - 6y = 0$ with the initial value problem $y(0) = 1$ and $y'(0) = -1$. (5 marks)

d) Find the **particular solution**, y_p of the second order non-homogeneous equation, $4y'' - 9y = -2x^2 - 3x - 18$. (7 marks)

[TOTAL 20 MARKS]

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APPENDIX

Derivatives: $f'(a)=\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$

Differentiation Rules

General Formulae

1. $\frac{d}{dx}[f(x)g(x)]=f(x)g'(x)+g(x)f'(x)$
2. $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]=\frac{g(x)f'(x)-f(x)g'(x)}{[g(x)]^2}$
3. $\frac{d}{dx}\left(x^n\right)=nx^{n-1}$
4. $\frac{d}{dx}[f(u)]=\frac{dy}{du}.\frac{du}{dx}$

Exponential and Logarithmic Functions

1. $\frac{d}{dx}\left(e^x\right)=e^x$
2. $\frac{d}{dx}(\ln x)=\frac{1}{x}$

Trigonometric Functions

1. $\frac{d}{dx}(\sin x)=\cos x$
2. $\frac{d}{dx}(\cos x)=-\sin x$
3. $\frac{d}{dx}(\tan x)=\sec^2 x$
4. $\frac{d}{dx}(\csc\ x)=-\csc\ x\cot\ x$
5. $\frac{d}{dx}(\sec x)=\sec x\tan x$
6. $\frac{d}{dx}(\cot x)=-\csc^2 x$

Table of Integrals

1. $\int u\ dv=uv-\int v\ du$
2. $\int u^n\ du=\frac{u^{n+1}}{n+1}+C,\quad n\neq -1$
3. $\int\frac{du}{u}=\ln|u|+C$
4. $\int e^u du=e^u+C$
5. $\int\sin u\ du=-\cos u+C$
6. $\int\cos\ u\ du=\sin u+C$
7. $\int\sec^2 u\ du=\tan u+C$
8. $\int\csc^2\ u\ du=-\cot\ u+C$
9. $\int\sec\ u\tan\ u\ du=\sec u+C$
10. $\int\csc u\cot u\ du=-\csc\ u+C$

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Differential Equations

Linear Differential Equations

$$\frac{dy}{dx} + p(x)y = q(x) \qquad \Rightarrow \qquad \mu y = \int \mu q(x)\,dx, \text{ where } \mu = e^{\int p(x)\,dx}$$

Constant Coefficient of Homogeneous Equations

Roots of Auxiliary Equation, $r = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

General Solutions to the Auxiliary Equation:

2 Real & Unequal Roots ($b^2 - 4ac > 0$)	$y = c_1 e^{r_1 x} + c_2 e^{r_2 x}$
Repeated Roots ($b^2 - 4ac = 0$)	$y = c_1 e^{rx} + c_2 x e^{rx}$
2 Complex Roots ($b^2 - 4ac < 0$)	$y = e^{ax}\left(c_1 \cos bx + c_2 \sin bx\right)$

Constant Coefficient of Non-Homogeneous Equations

$y = y_c + y_p$ [y_c: complementary solution, y_p: particular solution]

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